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ECE 520.435 Digital Signal Processing with MATLAB

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Problem Set 3

Problem 1: A causal LTI system is described by the following difference equation:

$$y(n) - \frac{3}{2}y(n-1) + \frac{1}{2}y(n-2) = x(n),\ n \geq 0$$

Determine:

1. the system function H(z)
2. plot the pole-zero plot in the z-plane
3. the unit impulse response h(n) and stem it
4. the unit step response v(n), that is, the response to the unit step u(n), and stem it
5. the frequency response function $H(e^{jw})$, and plot its magnitude and phase over $0 \leq w \leq \pi$
6. the system's response to input $x(n) = (\frac{1}{4})^n u(n)$ subject to $y(-1) = 4$ and $y(-2) = 10$

Problem 2: Determine the result of the following polynomial operations in MATLAB

1. $X_1(z) = (1 - 2z^{-1} + 3z^{-2} - 4z^{-3})(4 + 3z^{-1} - 2z^{-2} + z^{-3})$
2. $X_2(z) = (1 + z^{-1} + z^{-2})^3$
3. $X_3(z) = (z^{-1} - 3z^{-3} + 2z^{-5} + 5z^{-7} - z^{-9})(z + 3z^2 + 2z^3 + 4z^4)$
4. $X_4(z) = X_1(z)X_2(z) + X_3(z)$

Problem 3: Determine the inverse z-transforms using the partial fraction expansion method:

1. $X(z) = \frac{1-z^{-1}-4z^{-2}+4z^{-3}}{1-\frac{11}{4}z^{-1}+\frac{13}{8}z^{-2}-\frac{1}{4}z^{-3}}$. The sequence is right-sided
2. $X(z) = \frac{z}{z^3+2z^2+1.25z+0.25},\ |z| > 1$

Problem 4: For the LTI systems described below, determine

1. the impulse response representation

2. the pole-zero plot
3. the output $y(n)$ if the input is $x(n) = 3cos(\frac{\pi n}{3})u(n)$

(a) $H(z) = \frac{z^2-1}{(z-3)^2}$, anti-causal system

(b) $H(z) = \frac{z}{z-0.25} + \frac{1-0.5z^{-1}}{1+2z^{-1}}$, stable system

Problem 5: From Problem 1, decompose the solution $y(n)$ into:

1. transient response *Hint: response due to poles that are inside the unit circle*
2. steady-state response *Hint: response due to poles that are on the unit circle. If the poles are outside the unit circle, the response is termed an unbounded response*
3. homogeneous response *Hint: response due to system poles*
4. particular response *Hint: response due to input poles*
5. zero-state response *Hint:* $Y_{ZS}(z) = H(z)X(z)$
6. zero-input response *Hint:* $Y_{ZI}(z) = H(z)X_{IC}(z)$ *where* $X_{IC} = filtic(b, a, y_{IC}, x_{IC})$

Verify the results by computer the response mathematically. It will give you a good practice for your DSP Theory Exams on Z-Transforms.

Problem 6: A stable system has the following pole-zero locations:

$$z_1 = j,\ z_2 = -j,\ p_1 = -\frac{1}{2} + j\frac{1}{2},\ p_2 = -\frac{1}{2} - j\frac{1}{2}$$

It is also known that the frequency response function $H(e^{jw})$ evaluated at $w = 0$ is: $H(e^{j0}) = 0.8$. Determine:

1. Determine the system function $H(z)$ and indicate the ROC
2. Determine the difference equation representation

Problem 7: The **deconv(.)** function is useful in dividing two causal sequences. Write a code which *deconvolves*, or divide, two non-causal sequences (similar to the conv(.) function in Problem Set 2.

Hint: Deconvolution process is expounded on the next page

Deconvolution Deconvolution, as its name suggests, is the converse of the convolution process. It essentially involves solving (back-tracking) the convolution equation, i.e.

$$f * g = h$$

h (Don't confuse **h** with the impulse response. Here, **h** is just a dummy notation) can be thought of as a recorded signal (or measurement) and **f**, the actual signal (or measurement). The signal, **f**, has been modified (or shaped, or convolved) with another function **g**, where **g** may represent systems' transfer function, channel imperfections, noise and distortion, instrument contribution etc. The goal is to recover the original signal (**f**) from the recorded (or received, or measured) signal **h**. Let's see the convolution equation in transform domain (to be in context, I am using Z-domain):

$$F(z)G(z) = H(z)$$

Naturally, deconvolution, which can be done easily in transform domain, is:

$$G(z) = \frac{H(z)}{F(z)}$$

Deconvolution of one signal from another is achieved by dividing them in transform-domain. Deconvolution finds its applications in numerous realms like Signal and Image Processing, Seismology, Astronomy, Spectrometry etc. The most elementary example is to remove (or reverse) the distortion caused by electric filter.

function [p np r nr]=deconvolution(b,nb,a,na)
where
p is the polynomial part with support np=[np1 np2]
r is the remainder part with support nr=[nr1 nr2]
b is the numerator polynomial with support nb=[nb1 nb2]
a is the denominator polynomial with support na=[na1 na2]

Check your results with the convolution function you designed in Problem Set 2. MATLAB also incorporates a built-in function deconv(.) but, like conv(.), takes causal inputs only. Verify your result on the following operation:

$$\frac{z^2 + z + 1 + z^{-1} + z^{-2} + z^{-3}}{z + 2 + z^{-1}} = (z - 1 + 2z^{-1} - 2z^{-2}) + \frac{3z^{-2} + 3z^{-3}}{z + 2 + z^{-1}}$$